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CURRENT SWITCHING FROM A FAST TRIP INTO A SHUNT WIRE

V. A. Lebedev, D. P. Leper, and V. A. Yagnov

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In multistage current switching in a circuit containing an inductive energy store IES, the fast trip FT is shunted by one or more exploding wires [1-3]. To provide short times in the closing stage (on transferring the current to the load), it is desirable to perform the switching in the first stage at the maximum permissible current density in the shunting wire SW. We have to determine the maximum permissible current densities, since there are physical factors that prevent the transfer of a large current to a conductor of small cross section. So far, it has been considered that the main reason for failure at high current densities in the SW is breakdown in the FT at the stage when its electrical strength is recovering. In fact, as the density j_0 of the current switched into the wire increases, the time from the arc quenching in the FT to the explosion of the SW falls in proportion to j_0^{-2}, and therefore at high current densities the arc gap does not have time to recover its electrical strength by the instant of explosion, which leads to breakdown in the FT. However, we show here that the FT can fail also in an earlier stage, when the current transfer to the SW is not completed. The study is a theoretical consideration of the constraints in switching a current from an FT to SW arising from the rapid heating of the SW and the marked increase in resistance at high current densities.

1. The increase in resistance in the initial stages of electrical explosion in a conductor is [4, 5] determined in the main by the specific energy deposition $q = Q/m$, where $Q = \int_0^t i^2 R\,dt$ is the total deposited energy and m is the mass of the conductor. The increase in resistance is related to the energy input rate and is very marked at the stage of electrical explosion, but it does not play a large part in the initial stages (for $R/R_0 \leqslant 15$ for copper and for $R/R_0 \leqslant 11$ for aluminum). If the energy input rate to the conductor is small by comparison with the time for the phase transition from the solid state to the liquid one), then the dependence of the relative resistance on the specific energy deposition in

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the initial stage is quasistatic. At a high input rate, the relationship becomes linear and is a continuation of the linear relationship for the solid stage; the maximum deviation from the quasistatic relationship does not exceed 16% for copper (even less for aluminum). This is a quite sufficient accuracy for the switching model developed here, since the accuracy of the assumption of (2.1) on the linear increase in voltage in the FT can hardly exceed 15-20%. We use the quasistatic dependence of the relative resistance R/R_0 on the specific energy deposition q.

For the stage of heating in the solid state ($R < R_{mp}$, $q < q_{mp}$) we have as follows if we neglect the effect of thermal expansion:

$$R/R_0 = 1 + \alpha q/c_p = 1 + q/q_1, \tag{1.1}$$

where α is the temperature coefficient of resistance, c_p is the specific heat of the metal, and $q_1 = c_p/\alpha$ is the specific energy deposition corresponding to increase in specific resistance by a factor of two; $q_1 = 95$ J/g for copper, and $R_{mp}/R_0 = 5.96$, $q_{mp} = 468$ J/g. The following relationship [6] describes the melting stage on the assumption that the melting starts at the surface (the liquid and solid phases are considered as conductors connected in parallel):

$$\frac{R}{R_0} = \frac{R_{mp}}{R_0}\left\{1-\left[1-\frac{\rho_s}{\rho_l}\left(1+\frac{\Delta V}{V}\right)\frac{\Delta q}{\lambda}\right]\right\}^{-1}, \tag{1.2}$$

where ρ_s and ρ_l are the specific resistances of the solid and liquid phases at the melting point, λ and $\Delta V/V$ are the latent heat of fusion and the relative increase in volume on the phase transition, and $\Delta q = q - q_{mp}$. For copper $\rho_s/\rho_l = 0.48$, $\lambda = 0.21$ kJ/g, $\Delta V/V = 0.045$, and (1.2) is applicable for $5.96 \leqslant R/R_0 \leqslant 11.9$, 468 J/g $\leqslant q \leqslant$ 678 J/g. Equations (1.1) and (1.2) can be combined into a single relationship $R/R_0 = f(q/q_1) = f(Q/q_1 m)$. Differentiation of $f(Q/q_1 m)$ with respect to time gives the differential equation (3.2). The function $F(R/R_0)$ appearing in (3.2) is equal to one up to the melting point of the conductor ($R/R_0 \leqslant 5.96$).

2. In describing the arc in the FT, we use the model of (2.1) giving linear increase in the voltage, which is indicated by the following considerations. It is assumed that the extension of the arc is quasistationary, and then the arc is characterized by a statistical relationship between the field strength and current $E(i_a)$; the arc voltage is $U_a = E(i_a)vt$, where v is the rate of extension (subsequently assumed constant). For a blown arc working at 10^3-10^5 A under conditions of constant pressure, there is only a slight dependence of the field strength on the current, which enables one to replace $E(i_a)$ by some average value E. In that approximation, the voltage on the arc gap is

$$U_a = Evt. \tag{2.1}$$

In this description, the product Ev (the rate of increase in voltage) is a basic parameter characterizing the FT. For example, for the FT of explosive type examined in [7] $E \approx 100$ V/cm, $v \approx 10^5$ cm/sec, which gives $Ev \approx 10^7$ V/sec. The quasistationary extension of the arc no longer applied when i_a approaches zero; the field strength in the arc falls and the voltage deviates from the linear law of (2.1). A more detailed study can be made of this stage only by considering the dynamic properties of the arc [8, 9]. Note that the basic assumption of (2.1) is inapplicalbe to an FT in which the arc burns in a closed volume under conditions of increasing pressure.

An experimental test of (2.1) was based on measuring the voltage with an FT of explosive type working at a constant current i_0 (without shunting). In that case, the measured voltage is $U = U_a + i_0 dL_a/dt$, where L_a is the inherent arc inductance. The voltage waveforms (Fig. 1) for currents of 3 kA (solid line) and 25 kA (broken line) have a prominent linear part with $Ev \approx 1 \cdot 10^7$ V/sec and are very similar to one another. The error introduced by the $i_0 dL_a/dt$ term into the measured voltage is negligible. In fact, the arc inductance is $L_a \sim \mu_0 vt$, wnere $\mu_0 = 4\pi \cdot 10^{-7}$ H/m, $v = 10^5$ cm/sec, and at $i_0 = 25$ kA we have $i_0 dL_a/dt \sim i_0 \mu_0 v = 31$ V, which is only 1.3% of the maximum voltage of 2.3 kV developed on the FT.

3. The IES is the source the constant current i_0; the electrical circuit in switching from the FT to the SW contains two parallel branches: the FT branch described by (2.1) and the SW branch described by (3.2). Each of these branches has its own parasitic inductance L_t and L_s. The condition for equality of the voltages on the two branches gives

$$L_t di_a/dt + Evt = L_s\, di/dt + iR, \tag{3.1}$$

where R and i are the resistance and current for the SW branch. As $i_a = i_0 - i$, we get (3.2), where $L = L_t + L_s$ is the overall parasitic inductance.

The following is the complete system of equations describing the current switching from the FT to the SW:

$$L\,di/dt + iR = Evt; \tag{3.2}$$

$$\frac{1}{R_0}\frac{dR}{dt} = \frac{i^2 R}{q_1 m} F\left(\frac{R}{R_0}\right); \tag{3.3}$$

$$t = 0,\ i = 0,\ R = R_0. \tag{3.4}$$

The switching time τ_S in the SW is defined by the condition

$$i = i_0 \text{ for } t = \tau_S. \tag{3.5}$$

4. If we convert to dimensionless variables $x = t/\tau_1$, $y = i/i_1$, $z = R/R_0$, where $\tau_1 = \left[\frac{R_0 q_1 m}{(Ev)^2}\right]^{1/3}$; $i_1 = \left[\frac{Evq_1 m}{R_0^2}\right]^{1/3}$, then (3.2)-(3.5) become

$$\beta y' + yz = x; \tag{4.1}$$

$$z' = y^2 z F(z); \tag{4.2}$$

$$x = 0,\ y = 0,\ z = 1; \tag{4.3}$$

$$y = \gamma \quad \text{or} \quad x = \tau_K/\tau_1. \tag{4.4}$$

The dimensionless parameters β and γ appearing in (4.1) and (4.4) are defined by

$$\beta = L\left[\frac{(Ev)^2}{R_0^4 q_1 m}\right]^{1/3} = \frac{LS}{l^{5/3}}\left[\frac{(Ev)^2}{\rho_0^4 q_1 \delta}\right]^{1/3},$$

$$\gamma = i_0\left[\frac{R_0^2}{Evq_1 m}\right]^{1/3} = \frac{i_0 l^{1/3}}{S}\left[\frac{\rho_0^2}{Evq_1\delta}\right]^{1/3},$$

where $\delta = m/V$ is the density and ρ_0 is the initial specific resistance of the conductor (for copper $\delta = 8.9$ g/cm^3, $\rho_0 = 1.72 \cdot 10^{-6}$ $\Omega\cdot$cm), with l and S the length and cross section of the SW. Parameter β has the meaning of a dimensionless inductance, while γ is the dimensionless current density. States with identical values of β and γ are similar. The similarity is to be understood as similarity in the time dependence of the current for each of the branches, and also in the voltages on any of the circuit components and in the resistances of the arc and the SW, as well as the power due to the Joule losses in the arc and the SW.

5. In the limiting case of no inductance (L = 0), system (3.1)-(3.3) has an analytic solution that applies for the stage of heating in the solid state ($R/R_0 \leqslant 5.96$):

$$R = R_0\sqrt{1 + \frac{2}{3}\left(\frac{t}{\tau_1}\right)^3}, \quad i = \frac{Evt}{R_0\sqrt{1 + \frac{2}{3}\left(\frac{t}{\tau_1}\right)^3}}. \tag{5.1}$$

The function i(t) according to (5.1) has its maximum at $t_{max} = (3)^{1/3}\tau_1$, whose value is $i_{max} = (1/3)^{1/6} i_1$; switching is possible if $i_0 < i_{max}$, and this condition in dimensionless form is $\gamma < \gamma_* = (1/3)^{1/6} = 0.83$ and corresponds to there being a critical current density in the SW:

$$j_* = 0.83\left[Evq_1\delta/(\rho_0^2 l)\right]^{1/3}. \tag{5.2}$$

At current densities $j_0 = i_0/S > j_*$, the current in the SW passes through a maximum and begins to fall, and the current returns to the arc. Then (5.2) gives for a copper conductor that $j_* = 5.5 \cdot 10^4\ (Ev/l)^{1/3}$, where j_* is in A/cm^2, Ev is in V/sec, and l is in cm. The resistance for $t_{max} = 3^{1/3} \cdot \tau_1$ has the value $R_{max} = R_0\sqrt{3}$, which corresponds to a specific energy deposition of 70 J/g. The solution of (5.1) is approximately applicable when the voltage across the inductance Ldi/dt at any time is small by comparison with the voltage on the FT $U_a = Evt$. In that case we can neglect the term Ldi/dt in (3.1) and the term $\beta y'$ in (4.1). Switching with $|Ldi/dt| \ll U_a$ may be called noninductive or slow. The range of

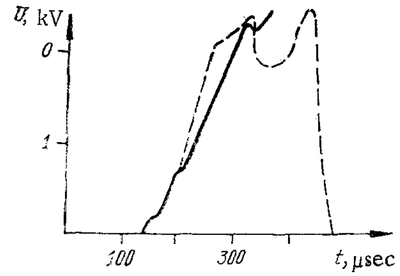

Fig. 1

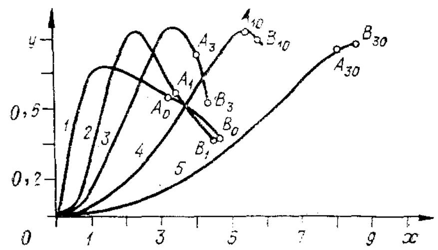

Fig. 2

applicability for the slow-switching approximation is shown by more detailed consideration to be defined by $\beta\gamma \ll 1$; as $\beta/\gamma = LS^2Ev/(i_0^2 l^2 \rho_0^2)$, the switching is slow if the inductance L is small, Ev is small, S is small, the length l is large, and the switched current i_0 is large.

6. The system (4.1)-(4.4) has been solved numerically by computer for the general case. Equations (4.1) and (4.2) together with the initial conditions (4.3) contain the parameter β, while parameter γ appears only in the dimensionless switching condition of (4.4). Therefore, (4.1)-(4.3) define a one-parameter family of solutions y(x, β), z(x, β). The y(x, β) curves (Fig. 2) show the time dependence of the current in the SW in terms of dimensionless variables, and there is a maximum for any β (β = 0, 1, 3, 10, and 30 for curves 1-5 correspondingly). The switching condition of (4.4) can be satisfied only for $\gamma < y_{max}(\beta)$; $\gamma > y_{max}(\beta)$ the current returns from the SW to the arc, which corresponds to failure in the FT. Therefore, y_{max} acts as the critical value γ_* corresponding to the boundary of the switching region. In Fig. 2, we have shown the points corresponding to the start of melting A_β and the point where all the mass of the conductor is in the liquid state B_β on the y(x, β) curves. Points A_β and B_β lie on the falling branch of the y(x, β) curve for β small, while for $\beta \gg 1$ the fall in current in the SW and the start of melting almost coincide in time.

Figure 3 shows the boundary of the switching region $\gamma_*(\beta)$ in the plane of β and γ, as well as R/R_0 = const curves corresponding to a constant relative increase in the resistance attained at the end of the switching. The values of R/R_0 are shown by the curves together with the corresponding values of the specific energy deposition. The boundary to the switching region for $\beta \ll 1$ lies at $\gamma_* = 0.83$, which corresponds to the noninductive limiting case. There is a slight increase in γ_* as β increases in the region $\beta \sim 1$ and then a slow fall in γ_* for $\beta \gg 1$ (approximately $\gamma_* \sim \beta^{-1/5}$). Figure 3 shows that γ_* varies very little (from 1 to 0.6) as β varies over a wide range, which means that we can replace the slow-switching condition $\beta/\gamma \ll 1$ near the boundary of the switching region (fast-switching condition $\beta/\gamma \gg 1$) by the simpler conditions $\beta \ll 1$ and $\beta \gg 1$; the curve R/R_0 = const for $\beta \gg 1$ is characterized by a fall in γ in accordance with the same law $\gamma \sim \beta^{-1/5}$. The curve $R/R_0 = 12$ begins at β = 54 and almost coincides with the switching boundary, which means that the conversion of the entire mass of conductor to the liquid state should almost never be observed at the end of switching.

Curve $\gamma = 0.8\gamma_*$ (broken line in Fig. 3) corresponds to a current density in the SW representing 80% of j_*; points 1-3 in Fig. 3 show points on the $\gamma = 0.8\gamma_*$ curve corresponding to switching of a current i_0 = 100 kA into a copper conductor with l = 100 cm with a voltage rise rate Ev = 10^6 V/sec (point 1), 10^7 V/sec (point 2), or 10^8 V/sec (point 3). The SW inductance is $L_s = 10^{-6}$ H and can be neglected. The cross sections of the SW for points 1-3 are correspondingly $L_t \approx 10^{-7}$ H and can be neglected. The cross sections of the

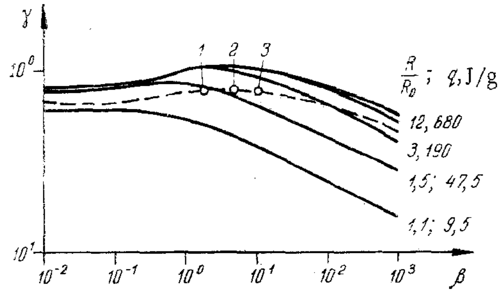

Fig. 3

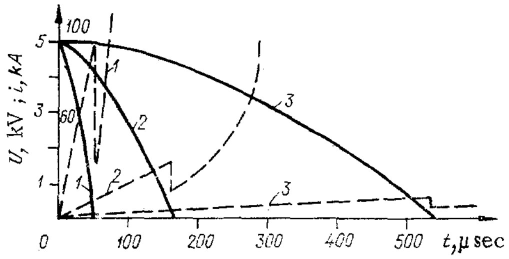

Fig. 4

SW for points 1-3 are correspondingly 8.5 mm^2, 3.8 mm^2, 1.91 mm^2. Figure 4 shows numerical waveforms for the currents in the FT (solid lines) and the voltages on the two-terminal FT-SW network corresponding to the parameters of points 1-3. The characteristic steps on the voltage oscillograms arise because the voltage on the inductance instantaneously becomes zero at the moments corresponding to the end of switching.

7. It has thus been shown that there are critical current densities j_* near which the heating of the conductor during the switching becomes important; for $j > j_*$ the resistance of the SW increases so rapidly that the current in the arc does not fall to zero and begins to rise again, i.e., the FT fails. With fast switching ($\beta \gg 1$), not more than 700 J/g can be pumped into the SW by the end of switching ($R/R_0 \approx 12$); with slow switching ($\beta \ll 1$), the permissible specific energy deposition is reduced to 70 J/g ($R/R_0 = \sqrt{3}$). The analytic expression for the critical current density in slow switching has been derived, while for fast switching the critical density may be calculated from Fig. 3 on the basis that

$$j_0 = \gamma[Evq_1\delta/(\rho_0 l)]^{1/3}.$$

We thus see that there are two possible reasons for failure of the FT at high current densities: 1) the return of the current to the arc considered here before completion of the current switching in the SW, and 2) breakdown in the arc gap at the stage of electrical strength recovery. It is at present uncertain which of these two constraints plays the main part. One assumes that under some conditions the first constraint will be the important one, while the second will be important under others. Our estimates for the critical current density in any case indicate an upper bound to the current density in the SW above which switching is impossible.

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THERMAL LIMITATION ON THE VELOCITY OF RING CONDUCTORS IN THE CASE OF INDUCTION AXIAL ACCELERATION

A. M. Baltakhanov and E. N. Ivanov UDC 538.54.001

The possibility of obtaining velocities of 1-5 km/sec with plane ring conductors in electromagnetic accelerators was demonstrated experimentally in [1, 2]. One of the main limitations in achieving high velocities when conductors are thrown into a magnetic field is the heating of the conductor and its transition from the solid state into a liquid or gaseous form. Nevertheless, in the practical utilization of high-velocity accelerators of macroparticles, the problem arises of determining the physical and mechanical properties and physical state of the thrown impactors. It is extremely difficult to solve this problem experimentally due to the short duration of the acceleration process and the high velocities of the impactors. In [3], by means of an approximate analysis, a relation is obtained which establishes the connection between the heating and the electromagnetic acceleration of the conductor, which holds over a range from the boiling point of nitrogen to the melting point of the corresponding metal, and in [4, 5] expressions are obtained for the limiting velocity of plane metal macroparticles in the ideal case of their acceleration in a uniform magnetic field.

In the present paper we consider the heating which occurs when a plane metal ring is accelerated in a two-dimensional pulsed magnetic field of a single-turn inductor.

The basic acceleration arrangement is shown in Fig. 1. The system of integrodifferential equations which describe the electromagnetic and electromechanical transients in this device has the form [6]

$$\tilde{\delta}(Q,t)+\frac{\mu_0\gamma(Q,t)}{2\pi}\frac{d}{dt}\sum_{i=1}^{2}\int\limits_{S_i}\tilde{\delta}(M,t)K(Q,M)\,ds_i=\begin{cases}\dfrac{\gamma(Q,t)}{2\pi\sqrt{r_Q}}\varphi[i_1(t)] & \text{for} \quad i=1,\\ 0 & \text{for} \quad i=2;\end{cases} \quad (1)$$

$$m\frac{dv}{dt}=\mu_0\int\limits_{s_2}\tilde{\delta}(Q,t)\sum_{i=1}^{2}\int\limits_{s_1}\tilde{\delta}(M,t)\frac{z_Q-z_M}{\sqrt{(z_Q-z_M)^2+(r_Q+r_M)^2}}\frac{1}{\sqrt{r_Mr_Q}}\left[-K+\frac{(z_Q-z_M)^2+r_Q^2+r_M^2}{(r_Q-r_M)^2+(z_Q-z_M)^2}E\right]ds_ids_2; \quad (2)$$

$$dz/dt=v, \quad (3)$$

where

$$\varphi[i_1(t)]=U_0-R_0i_1-L_0\frac{di_1}{dt}-\frac{1}{C}\int\limits_0^t i_1dt; \quad i_1=\int\limits_{s_1}\frac{\tilde{\delta}(M,t)}{\sqrt{r_M}}ds_1;$$

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